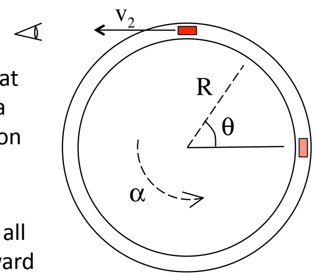
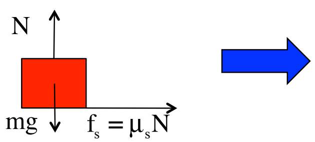
Problem 10.23



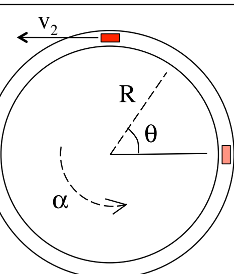
A car on a flat, circular track accelerates from rest at a tangential rate of $1.70\ \text{m/s}^2$. After having gone a quarter of the way around the track, it loses traction and slides out. What is the coefficient of friction between the tires and the track?

This is tricky. The temptation is to assume that all the frictional force is centripetal and acting toward the center of the track. The problem with that the only way the car can pick up speed is if it has some component of the frictional force pushing the car "forward." In other words, the f.b.d. and N.S.L. presentation shown below



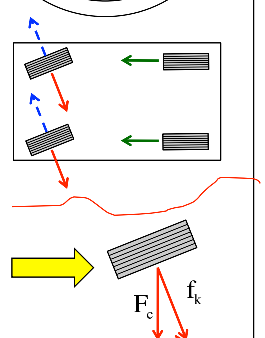
$$\sum F_x :$$
$$\mu_s N = m a_c$$

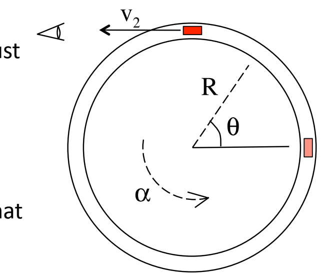
isn't an fully accurate representation of all we need here. (In fact, this is going to be a bit of a shaggy dog problem: translation—lots of pulling of oddball information together to get to the end.)



Static friction acts in *two ways* in an accelerating problem like this. It acts at the *back wheels* driving the car forward (the wheels apply a force to the ground throwing dirt backward; by N.T.L. the grounds applies a force on the tires accelerating them and the car forward). Friction additionally acts at the front tires when they are cranked into a turn (the tires push outward on the road while the road, in turn, pushes INWARD accelerating the car *out of* straight line motion--this inward push produces our centripetal force).

(Note: There is a subtlety to this turning force that is minor but interesting. The frictional force acts *perpendicular to the tires*, which means only a *component* of the force is centripetal. The other component acts opposite the direction of motion and slows the car down. The sketch shows the situation. If we assume the turning radius is large, then the wheels don't have to crank much and the frictional and centripetal forces approach one another. That is the assumption we will make here.)





Along the line of motion, the accelerating force is just the *mass* times the *tangential acceleration*, or:

$$F_t = m\left(1.70\ \text{m/s}^2\right)$$

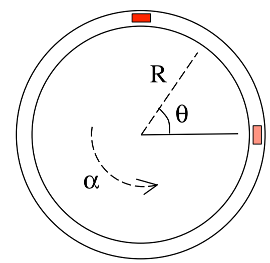
Just before the car breaks traction, the static centripetal frictional force is maximum. Knowing that

$$v^2/R = (R\omega)^2/R = R\omega^2,$$

we can write:

$$\begin{aligned} F_c &= m a_c \\ &= m\left(\frac{v_2^{\,2}}{R}\right) \\ &= m\left(\frac{(R\omega_2)^2}{R}\right) \\ &= mR\omega_2^{\,2} \end{aligned}$$

The vector sum of these two must equal to net frictional force acting on the car.



To get the angular speed at that point, we can use rotational kinematics to write:

$$\begin{aligned} (\omega_2)^2 &= (\cancelto{0}{\omega_1})^2 + 2\alpha\Delta\theta \\ &= 2\alpha\Delta\theta \end{aligned}$$

Using this, we can re-write the centripetal force expression as:

$$\begin{aligned} F_c &= mR\omega_2^{\,2} \\ &= mR\left(2\alpha\Delta\theta\right) \\ &= 2mR\Delta\theta\left(R\alpha\right) \\ &= 2mR\Delta\theta\left(a\right) \end{aligned}$$

Remembering that just before the car breaks traction, it has moved through an angular displacement of $\Delta\theta = 2\pi/4 = \pi/2$ radians, the net force acting on the car just before it breaks away is:

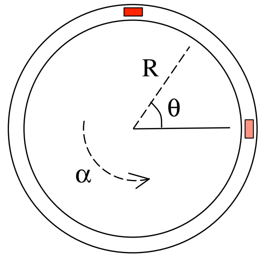
$$\begin{aligned} \vec{F}_{net} &= \left(\vec{F}_{cent}\right)(-\hat{r}) + \left(\vec{F}_t\right)\hat{\theta} \\ &= \left(\cancel{2}m\left(\pi/\cancel{2}\right)(a)\right)(-\hat{r}) + \left(m\left(1.70\ \text{m/s}^2\right)\right)\hat{\theta} \\ &= m\left[\left(\pi(a)\right)(-\hat{r}) + \left(1.70\ \text{m/s}^2\right)\hat{\theta}\right] \end{aligned}$$

The magnitude of this force is

$$\left|\vec{F}_{net}\right| = \left| m\left[(\pi(a))(-\hat{r}) + (1.70 \text{ m/s}^2)\hat{\theta} \right] \right|$$

$$= m\left[(\pi(a))^2 + (1.70 \text{ m/s}^2)^2 \right]^{1/2}$$

$$= m\left[\pi^2 a^2 + 2.89 \right]^{1/2}$$



As it is static friction, this force additionally has to equal $\mu_k N = \mu_k (mg)$. Soooo . . .

$$\mu_s (\cancel{m}g) = \cancel{m}\left[\pi^2 a^2 + 2.89 \right]^{1/2}$$

$$\Rightarrow \quad \mu_s = \frac{\left[\pi^2 (1.70 \text{ m/s}^2)^2 + (2.89 \text{ m}^2/\text{s}^4) \right]^{1/2}}{g}$$

$$= \frac{\left[\pi^2 (2.89 \text{ m}^2/\text{s}^4) + (2.89 \text{ m}^2/\text{s}^4) \right]^{1/2}}{(9.80 \text{ m/s}^2)}$$

$$= .572$$

Like I said, lots of odds and ends.